

# The puzzle of anomalously large isospin violations in $\eta(1405/1475) \rightarrow 3\pi$

Jia-Jun Wu<sup>1</sup>, Xiao-Hai Liu<sup>1</sup>, Qiang Zhao<sup>1,2\*</sup>, and Bing-Song Zou<sup>1,2†</sup>

1) *Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*

2) *Theoretical Physics Center for Science Facilities, CAS, Beijing 100049, China*

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The BES-III Collaboration recently report the observation of anomalously large isospin violations in  $J/\psi \rightarrow \gamma\eta(1405/1475) \rightarrow \gamma\pi^0 f_0(980) \rightarrow \gamma + 3\pi$ , where the  $f_0(980)$  in the  $\pi\pi$  invariant mass spectrum appears to be much narrower ( $\sim 10$  MeV) than the peak width ( $\sim 50$  MeV) measured in other processes. We show that a mechanism, named as triangle singularity (TS), can produce a narrow enhancement between the charged and neutral  $K\bar{K}$  thresholds, i.e.,  $2m_{K^\pm} \sim 2m_{K^0}$ . It can also lead to different invariant mass spectra for  $\eta(1405/1475) \rightarrow a_0(980)\pi$  and  $K\bar{K}^* + c.c.$ , which can possibly explain the long-standing puzzle about the need for two close states  $\eta(1405)$  and  $\eta(1475)$  in  $\eta\pi\pi$  and  $K\bar{K}\pi$ , respectively. The TS could be a key to our understanding of the nature of  $\eta(1405/1475)$  and advance our knowledge about the mixing between  $a_0(980)$  and  $f_0(980)$ .

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The spectrum of excited isoscalar states of  $J^{PC} = 0^{-+}$ , i.e. radial excitation states of  $\eta$  and  $\eta'$ , is still far from well-understood. An important and peculiar issue is about the nature of  $\eta(1405)$  and  $\eta(1475)$ . Although the Particle Data Group (PDG) [1] lists them as two individual states, there still exist unsolved puzzles in the understanding of their production and decay properties [2]. In fact, it is even still controversial whether they are two separated states or just one state of  $0^{-+}$  in different decay modes [2].

This situation is greatly improved with the availability of high-statistic  $J/\psi$  and  $\psi'$  events from the BES-III Collaboration. Very recently, the BES-III Collaboration report the observation of anomalously large isospin violations of the  $\eta(1405/1475) \rightarrow 3\pi$  in  $J/\psi \rightarrow \gamma\eta(1405/1475) \rightarrow \gamma\pi^0 f_0(980) \rightarrow \gamma + 3\pi$  [3]. What makes this measurement extremely interesting is its involvement with the  $a_0(980)$ - $f_0(980)$  mixings, which has also been a long-standing puzzle in history. In Ref. [3], it was shown that there is only one enhancement in the vicinity of 1.44 GeV, which corresponds to the  $\eta(1405)$  or  $\eta(1475)$  state. Moreover, it shows that the  $f_0(980)$  signal is only about 10 MeV in width, and the preliminary branching ratio for the isospin-violating decay  $\eta(1405/1475) \rightarrow f_0(980)\pi^0$  turns out to be significant.

At a first glance, this process seems to be complicated by the correlations between the  $\eta(1405/1475)$  ambiguity and  $a_0(980)$ - $f_0(980)$  mixings. However, we shall show in this work, this process would provide a golden opportunity to disentangle the relation between the  $\eta(1405)$  and  $\eta(1475)$  signal, and identify a crucial dynamic mechanism in the isospin-violating decay of  $\eta(1405/1475) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$ , i.e. the triangle singularity (TS), where the intermediate on-shell  $K\bar{K}^* + c.c.$  pair can exchange an

on-shell kaon, and then rescatter to the isospin violating  $f_0(980)\pi^0$ .

For this purpose, it is necessary to study the decay of  $\eta(1405/1475) \rightarrow a_0(980)\pi \rightarrow \eta\pi\pi$  as a correlated process with  $\eta(1405/1475) \rightarrow f_0(980)\pi^0 \rightarrow 3\pi$ . Furthermore, since the TS mechanism is driven by the tree-level process  $\eta(1405/1475) \rightarrow K\bar{K}^* + c.c.$ , we shall show that only one  $0^{-+}$  isoscalar state is needed here. This “one state” assumption is our starting point of this work. To keep the notation short, we denote it as  $\eta(1440)$  in the following analysis.

In this letter, we shall demonstrate that the TS mechanism can lead to different mass spectra for  $\eta(1440) \rightarrow K\bar{K}^* + c.c.$  and  $a_0(980)\pi^0$ . Namely, due to the TS, the  $\eta(1440)$  mass peaks in  $K\bar{K}^* + c.c.$  and  $a_0(980)\pi$  would appear differently. Moreover, anomalously large isospin violations may occur in  $\eta(1440) \rightarrow f_0(980)\pi^0$ . These features as a consequence of the TS mechanism could be a natural solution for the long-standing puzzle about the nature of  $\eta(1405/1475)$  in experimental analyses.

As follows, a coherent investigation of these three transitions,  $\eta(1440) \rightarrow K\bar{K}\pi$ ,  $\eta\pi\pi$  and  $3\pi$ , is presented. In Figs. 1, 2 and 3, their decays are illustrated by the schematic Feynman diagrams, respectively. These transitions are studied in an effective Lagrangian approach with the vector-vector-pseudoscalar (VVP), vector-pseudoscalar-pseudoscalar (VPP), and scalar-pseudoscalar-pseudoscalar (SPP) couplings as the following:

$$L_{V_1 V_2 P} = g_{V_1 V_2 P} \varepsilon_{\alpha\beta\mu\nu} p_{V_1}^\alpha p_{V_2}^\beta \psi_{V_1}^\mu \psi_{V_2}^\nu \psi_P, \quad (1)$$

$$L_{V P_1 P_2} = g_{V P_1 P_2} (\psi_{P_1} \partial_\mu \psi_{P_2} - \psi_{P_2} \partial_\mu \psi_{P_1}) \psi_V^\mu, \quad (2)$$

$$L_{S_1 P_2 P_3} = g_{S_1 P_2 P_3} \psi_{S_1} \psi_{P_2} \psi_{P_3}, \quad (3)$$

where  $\psi_V$ ,  $\psi_P$  and  $\psi_S$  stand for the vector, pseudoscalar, and scalar fields, respectively. The following relations based on the SU(3) flavor symmetry are implied:  $g_{f K \bar{K}} = g_{f K^+ K^-} = -g_{f K^0 \bar{K}^0}$ ,  $g_{a K \bar{K}} = g_{a K^+ K^-} = g_{a K^0 \bar{K}^0}$ ,  $\sqrt{2}g_{f \pi \pi} = \sqrt{2}g_{f \pi^0 \pi^0} = -g_{f \pi^+ \pi^-}$ ,  $g_{\eta K^* \bar{K}} =$

\*zhaoq@ihep.ac.cn

†zoubs@ihep.ac.cn

$$\begin{aligned}
g_{\eta K^*+K^-} &= -g_{\eta K^*-K^+} = -g_{\eta K^*0\bar{K}^0} = g_{\eta\bar{K}^*0K^0}, \\
g_{K^*K\pi}^c &= g_{K^*+K+\pi^0} = -g_{K^*-K-\pi^0}, \text{ or } g_{K^*K\pi}^n = \\
&= -g_{K^*0K^0\pi^0} = g_{\bar{K}^*0\bar{K}^0\pi^0}.
\end{aligned}$$

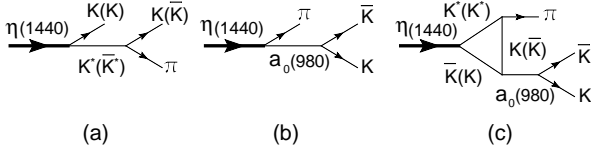


FIG. 1: The schematic diagrams for  $\eta(1440) \rightarrow K\bar{K}\pi$ . Diagram (a) and (b) denotes the tree-level transitions, while (c) illustrates the transition via the TS mechanism.

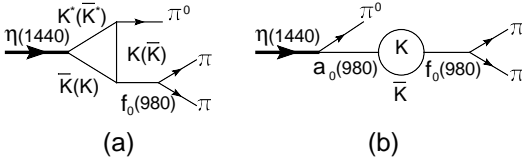


FIG. 2: The schematic diagrams for  $\eta(1440) \rightarrow 3\pi$ . Diagram (a) is driven by the TS mechanism, while (b) gives contributions from the  $a_0(980)$ - $f_0(980)$  mixing.

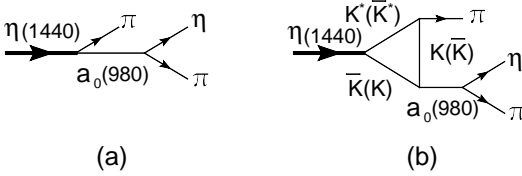


FIG. 3: The schematic diagram for  $\eta(1440) \rightarrow \eta\pi\pi$  via (a) the tree-level  $a_0(980)\pi$  production and (b) the TS mechanism.

The coupling constants of  $a_0(980)$  come from Ref.[4]:  $g_{aK\bar{K}} = 3.33$  GeV and  $g_{a\eta\pi} = 2.45$  GeV. KLOE [5] and BES [6] give different values for  $f_0(980)$ , i.e. KLOE:  $g_{fK\bar{K}} = 5.92$  GeV, and  $g_{f\pi\pi} = 2.09$  GeV; BES:  $g_{fK\bar{K}} = 4.18$  GeV, and  $g_{f\pi\pi} = 1.66$  GeV. Fortunately, these two groups of parameters give almost the same results when the Flatte form of propagator is applied to  $f_0(980)$ . In other words, the values of the coupling product  $g_{fK\bar{K}}g_{f\pi\pi}G_f$  are almost the same. The coupling constant  $g_{K^*K\pi}$  is calculated from the width of  $K^* \rightarrow K\pi$  [1], i.e.  $g_{K^*K\pi}^c = 3.268$  for the charged channel, and  $g_{K^*K\pi}^n = 3.208$  for the neutral channel.

We note that energy-dependent widths are adopted for the  $a_0(980)$  and  $f_0(980)$  propagators:

$$\begin{aligned}
G_a &= 1/[s - m_f^2 + i\sqrt{s}\Gamma_a(s)], \\
G_f &= 1/[s - m_a^2 + i\sqrt{s}\Gamma_f(s)],
\end{aligned} \quad (4)$$

where

$$\Gamma_a(s) \equiv \{g_{aK\bar{K}}^2[\rho(K^0, \bar{K}^0) + \rho(K^+, K^-)] + g_{a\eta\pi}^2\rho(\pi^0, \eta)\} / (16\pi\sqrt{s}), \quad (5)$$

$$\Gamma_f(s) \equiv \{g_{fK\bar{K}}^2[\rho(K^0, \bar{K}^0) + \rho(K^+, K^-)] + g_{f\pi\pi}^2[\rho(\pi^0, \pi^0) + 2\rho(\pi^+, \pi^-)]\} / (16\pi\sqrt{s}) \quad (6)$$

with  $\rho(A, B) \equiv \frac{1}{2s}[(s - (m_A + m_B)^2)(s - (m_A - m_B)^2)]^{1/2}$ .

To compare with the experimental measurement and take into account the width effects from the  $\eta(1440)$  in the typical decay  $J/\psi \rightarrow \gamma\eta(1440) \rightarrow \gamma ABC$ , the following standard expression is adopted,

$$\begin{aligned}
&\frac{d\Gamma_{J/\psi \rightarrow \gamma\eta(1440) \rightarrow \gamma ABC}}{d\sqrt{s_0}} \\
&= \frac{2s_0}{\pi} \frac{\Gamma_{J/\psi \rightarrow \gamma\eta(1440)}(s_0) \times \Gamma_{\eta(1440) \rightarrow ABC}(s_0)}{(s_0 - m_{\eta(1440)}^2)^2 + \Gamma_{\eta(1440)}^2 m_{\eta(1440)}^2} \quad (7)
\end{aligned}$$

where  $s_0$  is the four-momentum square of  $\eta(1440)$  in the reaction, and  $\Gamma_{\eta(1440)}$  can be  $s_0$ -dependent:

$$\Gamma_{\eta(1440)}(s_0) = \Gamma_{\eta(1440) \rightarrow K\bar{K}^* + c.c. \rightarrow K\bar{K}\pi}(s_0), \quad (8)$$

which is the tree-level contribution from Fig. 1(a). We note that the contribution of Fig. 1(b) via  $a_0(980)\pi$  is negligibly small in comparison with Fig. 1(a).

Old data for  $J/\psi \rightarrow \gamma\eta(1405/1475) \rightarrow \gamma K\bar{K}\pi$  were available from DM2 [7], MARK III [8], and BES-I [9, 10]. Although there have been reports of simultaneous observation of two pseudoscalars around 1.44 GeV, as reviewed by the PDG2010 [1], the presence of two states still needs confirmation. Note that most observations of the  $\eta(1405)$  and  $\eta(1475)$  are in different decay channels with different masses. Thus, we first investigate the invariant mass spectrum of  $K\bar{K}\pi$  in  $J/\psi \rightarrow \gamma\eta(1440) \rightarrow \gamma K\bar{K}\pi$  by fitting the experimental data [7–10]. We then fix the fitted mass  $m_{\eta(1440)}$  and coupling  $g_{\eta(1440)K^*\bar{K}}$ , and apply them to  $\eta(1440) \rightarrow 3\pi$  and  $\eta\pi\pi$ .

The fit of the invariant mass spectrum of  $K\bar{K}\pi$  is shown in Fig. 4. The thick solid line illustrates the best fit by a constant width  $\Gamma_{\eta(1440)} = 67$  MeV at  $m_{\eta(1440)} = 1.42$  GeV. The thick dashed line denotes the fit with an energy-dependent width  $\Gamma_{\eta(1440)}(\sqrt{s_0}) = 166$  MeV at  $\sqrt{s_0} = 1.42$  GeV and  $m_{\eta(1440)} = 1.55$  GeV. It is interesting to see that all those three data sets are fitted well, and the peak position is about 1.44 GeV. The background is estimated by a polynomial form as shown by the thin lines.

In Figs. 2 and 3, the  $K\bar{K}^*(K)$  loops, i.e. the TS mechanism, play a dominant role in both  $\eta(1440) \rightarrow 3\pi$  and  $\eta\pi\pi$ . For  $\eta(1440) \rightarrow 3\pi$ , the  $a_0(980)$ - $f_0(980)$  mixing can also contribute but is relatively small. We discuss the key features of these transitions as follows:

i) In Fig. 2(b), the contribution from the  $a_0(980)$ - $f_0(980)$  mixing can be estimated by the  $K\bar{K}$  loops,

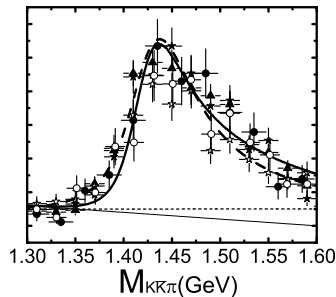


FIG. 4: The fit for invariant mass spectrum of  $K\bar{K}\pi$  with a constant (thick solid) or energy-dependent (thick dashed line) width. The solid triangles, solid circles, hollow circles, solid pentacles, and hollow pentacles are experimental data from MARK III ( $K_S^0 K^\pm \pi^\mp$ ) [8], BES-I ( $K_S^0 K^\pm \pi^\mp$ ) [9], BES-I ( $K^+ K^- \pi^0$ ) [10], DM2 ( $K_S^0 K^\pm \pi^\mp$ ) [7], and DM2 ( $K^+ K^- \pi^0$ ) [7], respectively.

where the incomplete cancellation between the charged and neutral  $K\bar{K}$  loops (due to the mass difference between the charged and neutral kaons) will lead to non-vanishing transition matrix element between  $a_0(980)$  and  $f_0(980)$ . The predicted mixing intensities for  $f_0(980) \rightarrow a_0(980)$  [11] and  $a_0(980) \rightarrow f_0(980)$  [12] turn out to be consistent with the BES-III measurements [13] in  $J/\psi \rightarrow \phi\eta\pi^0$  and  $\chi_{c1} \rightarrow 3\pi$ , respectively.

Given that the  $a_0(980)$ - $f_0(980)$  mixing is the exclusive mechanism in a typical transition of  $X \rightarrow Y a_0^0(980) \rightarrow Y f_0(980) \rightarrow Y \pi\pi$ , its ratio to the corresponding tree-level process  $X \rightarrow Y a_0^0(980) \rightarrow Y \eta\pi^0$  is about  $10^{-2}$ . This allows us to estimate the magnitude of Fig. 2(b) in respect of experimental branching ratio of  $3 \times 10^{-4}$  for  $J/\psi \rightarrow \gamma\eta(1405/1475) \rightarrow \gamma\eta\pi^+\pi^-$  [1], of which the upper limit is about  $10^{-6}$ . This value is nearly one order of magnitude smaller than the experimental data of  $10^{-5}$  [3]. This is a rather direct indication that only the  $a_0(980)$ - $f_0(980)$  mixing is not sufficient for accounting for the new data for  $J/\psi \rightarrow \gamma\eta(1440) \rightarrow \gamma + 3\pi$ .

ii) The mechanism of Fig. 2(a) is very different from the  $a_0$ - $f_0$  mixing scheme. It not only produces strong isospin violations in the subprocess  $\eta(1440) \rightarrow 3\pi$ , but also enhances such a breaking by the TS mechanism, i.e. within a particular kinematic region, all those three internal particles ( $K\bar{K}^*K$ ) are allowed to be on-shell.

Note that a full integral of the  $K\bar{K}^*(K)$  loop has ultraviolet divergence. Thus, an empirical form factor generally has to be introduced in the numerical calculation of the loop amplitudes, and model-dependence seems to be inevitable. However, it should be recognized that the absorptive part of the loop integral is rather model-independent here. Because of the presence of the on-shell kinematics for all those internal particles, i.e. the TS mechanism, the absorptive part of the loop integral can be calculated directly in the on-shell approximation.

In Fig. 5, the absorptive and dispersive part of the loop integrals are calculated using different cut-off schemes. We skip the details of form factors adopted here, but just point out that the absorptive parts of the charged and neutral loops are predominantly driven by the TS, and insensitive to the form factors. In Fig. 5(a) the peaking positions of the absorptive amplitudes for the charged and neutral loops in the  $K\bar{K}$  invariant mass spectrum are given by the thresholds of  $K^0\bar{K}^0$  and  $K^+K^-$  due to the TS condition. In contrast, we find a sensitivity of the dispersive part to the form factors as shown by Fig. 5(b). Interestingly, it shows that the cancellation between the charged and neutral amplitude is still insensitive to the form factor. This indicates a model-independent feature of the isospin-violating amplitude via the TS mechanism. Moreover, we confirm the dominance of the TS mechanism (Fig. 2(b)) in  $J/\psi \rightarrow \gamma\eta(1440) \rightarrow \gamma + 3\pi$ . For the isospin-conserved transition,  $J/\psi \rightarrow \gamma\eta(1440) \rightarrow \gamma\eta\pi\pi$ , the dispersive part becomes strongly model-dependent since it is given by the sum of the charged and neutral amplitudes.

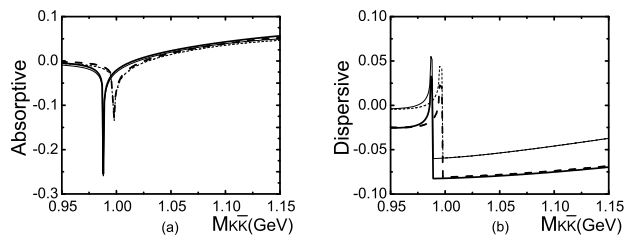


FIG. 5: The absorptive (a) and dispersive part (b) of the loop amplitude versus the invariant mass of  $K\bar{K}$  at  $\sqrt{s_0} = 1.42$  GeV for the  $\eta(1440)$ , with solid and broken curves for the charged and neutral  $K\bar{K}^*$  loops, respectively.

iii) Because of the dominance of TS mechanism in Figs. 2 and 3, it predicts a narrow peak around the  $f_0(980)$  in the invariant mass spectrum of  $\pi^+\pi^-$  due to the cancellation between the charged and neutral  $K\bar{K}^*(K)$  loops, while the  $a_0(980)$  in the  $\eta\pi^0$  invariant mass spectrum is not necessarily narrow. Note that the  $f_0(980)$  has a peak width about 50 MeV [1], a much narrower structure ( $\sim 10$  MeV) around  $f_0(980)$  as demonstrated in Fig. 6 is an indisputable signature for the TS mechanism.

iv) In Fig. 7, the invariant mass spectra of  $\eta\pi^0\pi^0$  and  $\pi^+\pi^-\pi^0$  are plotted with the same pole mass (1.42 GeV) and width as in  $\eta(1440) \rightarrow K\bar{K}\pi$ . Interestingly, the peak position is obviously shifted by the TS terms, and appears to be about 1.415 GeV. Recall that in Fig. 4 the peak position is around 1.44 GeV. It shows that the TS mechanism should be responsible for the observed mass difference for  $\eta(1405/1475)$  in the  $\eta\pi^0\pi^0$  and  $K\bar{K}\pi$  spectrum. It is worth mentioning that the invariant mass spectrum of  $\pi^+\pi^-\pi^0$  is totally independent of the cut-off

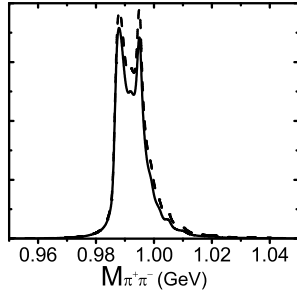


FIG. 6: The invariant mass spectrum of  $\pi^+\pi^-$  from the TS mechanism in Fig. 2(b). The solid line is given by calculations with a constant width for  $\eta(1440)$ , while the dashed line is with an energy-dependent width.

energy of the form factor since the difference of charged and neutral loop contributions is basically independent of the cut-off parameter. This TS mechanism also gives correctly the relative production rates for  $\eta(1440) \rightarrow K\bar{K}\pi$ ,  $\eta\pi\pi$  and  $\pi^+\pi^-\pi^0$ .

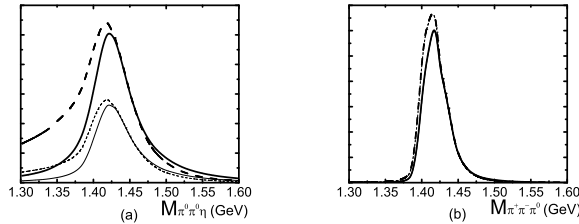


FIG. 7: The invariant mass spectra of (a)  $\eta\pi^0\pi^0$  and (b)  $\pi^+\pi^-\pi^0$ . The solid lines are constant-width calculations for  $\eta(1440)$ , while the dashed lines are energy-dependent-width calculations. The thick and thin lines in (a) and (b) are results using different cut-off energies in the form factor, respectively.

To summarize, we have studied the  $J/\psi \rightarrow \gamma\eta(1405/1475)$  decays into three different final states:  $K\bar{K}\pi$ ,  $\pi^+\pi^-\pi^0$  and  $\eta\pi^0\pi^0$ . By the assumption that only one state of  $0^{-+}$  is present around 1.44 GeV, i.e.  $\eta(1440)$ , our results are in good agreement with the experimental data. The dynamic reason is because of the TS mechanism, which allows all the internal particles to be on-shell in this particular kinematic region. This mechanism turns out to be much more dominant than the  $a_0$ - $f_0$  mixing term, and can thus lead to the anomalously large isospin violations in  $\eta(1440) \rightarrow \pi^+\pi^-\pi^0$ . Nevertheless, the peak positions of the  $\eta(1440)$  in the invariant mass spectra of  $K\bar{K}\pi$ ,  $\pi^+\pi^-\pi^0$  and  $\eta\pi^0\pi^0$  are shifted

by the TS amplitudes and appear to have different line-shapes. This phenomenon can possibly explain the puzzling presence of two states  $\eta(1405)$  and  $\eta(1475)$  in the previous data analysis, and improve our understanding of the isoscalar pseudoscalar spectrum [14]. This clarification that only one state  $\eta(1440)$  is eventually needed in this energy region is nontrivial since it could also provide important insights into the lightest pseudoscalar glueball candidate, which could be much heavier than  $\eta(1440)$  as predicted by the lattice QCD [15]. Further relevant issues can be studied by BES-III experiment in the near future.

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